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GEOMETRICALLY NON-LINEAR ANALYSIS OF A THICK ANNULAR PLATE WITH ELASTICALLY CONSTRAINED EDGE USING GALERKIN'S METHOD

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1. INTRODUCTION

Raju and Rao [1] and Reddy and Huang [2] have presented axisymmetric non-linear vibration of moderately thick circular and annular plates using the finite-element method. Galerkin solutions using one-term spatial mode shape for deflection, are presented by Sathyamoorthy [3, 4] for non-linear vibration of circular thick plates, employing first order shear deformation theory and Berger's approximation. Dumir and Shingal [5, 6] have presented geometrically non-linear static, transient and postbuckling analysis of thick annular plates, employing orthogonal point collocation method. Neetha *et al.* [7] have developed postbuckling analysis of a moderately thick elastic annular plate using the finite-element method.

This work presents an approximate closed-form analytical solution of the geometrically non-linear, axisymmetric, moderately large deflection response of a polar orthotropic, moderately thick, annular plate with a free hole and elastically constrained outer edge. Neglecting the rotational and in-plane inertia, the geometrically non-linear first order shear deformation theory (FSDT) is formulated in terms of deflection w, rotation ψ of the normal to the midplane and the stress function Φ. The rotation ψ is approximated by a one-term mode shape and the moment equation of equilibrium is solved exactly for w satisfying all out-of-plane boundary conditions. The compatibility equation is solved exactly for Φ satisfying the in-plane boundary conditions. The Galerkin's method is applied to the equation of motion to obtain Duffing-type equation for the deflection. The effect of the rotational and in-plane stiffnesses of the support and the thickness and orthotropic parameters on the buckling, postbuckling, static, transient and non-linear vibration response of orthotropic thick plates is studied.

2. GOVERNING EQUATIONS OF FSDT

Consider an axisymmetric deformation of an annulus of outer radius a, inner radius b, thickness h and density γ, with mid-plane at $z = 0$. For a plate with elastically restrained outer edge, with rotational and inplane stiffnesses $\bar{k}_b$, $\bar{k}_i$, subjected to applied in-plane radial force resultant $\bar{N}$ and uniformly distributed transverse load q, the boundary conditions at $r = a$ are

$$\bar{w} = 0, \qquad M_r = -\bar{k}_b\bar{\psi}, \quad N_r = \bar{\Phi}/r = \bar{N} - \bar{k}_i\bar{u}^0, \tag{1}$$

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where $\bar{w}$, $\bar{u}^0$ and $\bar{\psi}$ are the transverse and radial displacements of the mid-plane, rotation of its normal and $\bar{\Phi}$ is the stress function. The simply supported (S), clamped (C) and movable (M), immovable (I) boundary conditions correspond to $\bar{k}_b = 0$, ∞ and $\bar{k}_i = 0$, ∞ respectively. The variables are non-dimensionalized as

$$\rho = r/a, \qquad w = \bar{w}/h, \qquad u = \bar{u}a/h^2, \qquad \psi = a\bar{\psi}/h, \qquad \Phi = a\bar{\Phi}/E_0 h^3,$$

$$\tau = t(h/a^2)(E_0/\gamma)^{1/2}, \qquad A^*_{\alpha\beta} = \bar{A}^*_{\alpha\beta}E_0 h, \qquad A_{55} = \bar{A}_{55}/E_0 h, \qquad D_{\alpha\beta} = \bar{D}_{\alpha\beta}/E_0 h^3,$$

$$s = a/h, \qquad \xi = b/a, \qquad N = \bar{N}a^2/E_0 h^3, \qquad Q = qa^4/E_0 h^4,$$

$$k_i = \bar{k}_i a/E_0 h, \qquad k_b = \bar{k}_b a/E_0 h^3, \tag{2}$$

where E_0 is a specific Young's modulus, $(\bar{A}, \bar{D}) = (h, h^2/12)\bar{Q}$, $\bar{A}^* = \bar{A}^{-1}$ and $\bar{A}_{55} = k_5^2 h G_{zr}$, with $[\bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}] = [E_r, \nu_{r\theta}E_\theta, E_\theta]/(1 - \nu_{r\theta}\nu_{\theta r})$. E_i are Young's moduli, ν_{ij} are the Poisson ratios and G_{ij} are shear moduli. The shear correction factor k_5^2 is taken as 5/6.

The dimensionless forms of the governing equations of the geometrically non-linear FSDT [8] are

$$A^*_{22}(\rho\Phi'' + \Phi') - A^*_{11}\Phi/\rho = -w'^2/2, \tag{3}$$

$$D_{11}(\psi'' + \psi'/\rho) - D_{22}\psi/\rho^2 - s^2 A_{55}(\psi + w') = 0, \tag{4}$$

$$[s^2 A_{55}\rho(\psi + w') + \Phi w']'/\rho + Q - \ddot{w} = 0, \tag{5}$$

$$\psi(\xi) + w'(\xi) = 0, \quad D_{11}\psi'(\xi) + D_{12}\psi(\xi)/\xi = 0, \quad D_{11}\psi'(1) + D_{12}\psi(1) = -k_b\psi(1), \tag{6}$$

$$w(1) = 0, \quad \Phi(\xi) = 0, \quad \Phi(1) = N - k_i[A^*_{21}\Phi(1) + A^*_{22}\Phi'(1)], \tag{7}$$

where $(\ )' = (\ )_{,\rho}$, $(\dot{\ }) = (\ )_{,\tau}$.

3. METHOD OF SOLUTION

The approximate analytical solution is based on assumed spatial mode shapes for ψ, w having the same form as in the linear solution for static load Q. The rotation ψ is approximated as

$$\psi = \eta(\tau)\sum_{i=1}^{4}\psi_i\rho^{n_i} \tag{8}$$

with $n_i = 3, 1, -p, p$; $p = (D_{22}/D_{11})^{1/2} = (A^*_{11}/A^*_{22})^{1/2}$, $\psi_4 = 1$. ψ_1, ψ_2 and ψ_3 are obtained from equations (6) using equation (4):

$$(9D_{11} - D_{22})\xi\psi_1 + (D_{11} - D_{22})\psi_2/\xi = 0,$$

$$G_3(\xi)\psi_1 + G_1(\xi)\psi_2 + G_{-p}(\xi)\psi_3 = -G_p(\xi),$$

$$[G_3(1) + k_b]\psi_1 + [G_1(1) + k_b]\psi_2 + [G_{-p}(1) + k_b]\psi_3 = -k_b - G_p(1), \tag{9}$$

with $G_m(\rho) = (mD_{11} + D_{12})\rho^{m-1}$. Equation (4) is solved accurately, subject to condition $(7)_1$, in order to yield

$$w = \eta(\tau)\left[\sum_{i=1}^{5} w_i \rho^{m_i} + w_6 \ln\rho\right], \quad w' = \eta(\tau)\sum_{i=1}^{5} d_i \rho^{n_i} \tag{10}$$

with $m_i = 0, 4, 2, 1-p, \;\; 1+p, \;\; w_2 = -\psi_1/4, \;\; w_3 = -\psi_2/2 + (9D_{11} - D_{22})\psi_1/2s^2A_{55}$, $w_4 = \psi_3/(p-1)$, $w_5 = -1/(p+1)$, $w_6 = (D_{11} - D_{22})\psi_2/s^2A_{55}$,

$$w_1 = -w_2 - w_3 - w_4 - w_5, \qquad d_i = m_{i+1}w_{i+1}, \quad i = 1,\ldots,4, \quad d_5 = w_6, \quad n_5 = -1. \tag{11}$$

Equation (3) is solved accurately, subject to conditions $(7)_2$ and $(7)_3$, in order to yield the stress function Φ as

$$\Phi = \eta^2(\tau)\left[\sum_{i=1}^{5}\sum_{j=1}^{5*} d_i d_j I_{n_i+n_j+1}\rho^{n_i+n_j+1} + \sum_{m=1}^{2}(a_k + c_k\ln\rho)\rho^{l_k}\right] + N\sum_{m=1}^{2} b_k\rho^{l_k} \tag{12}$$

with $l_1 = p$, $l_2 = -p$ and

$$[a_1, a_2, b_1, b_2] = [(x_2x_6 - x_4x_5), (x_3x_5 - x_1x_6), x_4, \; -x_3]/(x_1x_4 - x_2x_3),$$

$$[c_1, c_2] = [-d_4, d_3]d_5/2pA^*_{22}, \quad x_1 = 1 + k_i(A^*_{21} + pA^*_{22}), \quad x_2 = 1 + k_i(A^*_{21} - pA^*_{22}),$$

$$x_3 = \xi^p, \quad x_4 = \xi^{-p},$$

$$x_5 = \sum_{i=1}^{5}\sum_{j=1}^{5*}[1 + k_i\{A^*_{21} + (n_i + n_j + 1)A^*_{22}\}]d_i d_j I_{n_i+n_j+1} + k_i A^*_{22}(c_1 + c_2),$$

$$x_6 = \sum_{i=1}^{5}\sum_{j=1}^{5*} d_i d_j I_{n_i+n_j+1}\xi^{n_i+n_j+1} + \sum_{m=1}^{2} c_m\xi^{l_m}\ln\xi, \quad I_q = -1/2(q^2A^*_{22} - A^*_{11}). \tag{13}$$

The superscript $*$ in the double sum denotes that the terms for $(i, j) = (3,5), (4,5), (5,3), (5,4)$ are excluded. Substituting ψ, w, Φ from equations (8), (10) and (12) into equation (5), applying the Galerkin procedure of multiplying by ρw and integrating from $\rho = \xi$ to $\rho = 1$, yields on integration by parts, an equation for η:

$$B_1\ddot{\eta} + (B_2 + B_3N)\eta + B_4\eta^3 = B_5Q, \tag{14}$$

where

$$B_1 = -\sum_{i=1}^{5}\hat{J}_{m_i+2}w_i - [\bar{L}_2 - \hat{J}_2 - \xi^2 J_0\ln\xi]w_6/2,$$

$$B_2 = (9D_{11} - D_{22})\psi_1 J_2 + (D_{11} - D_{22})\psi_2 J_0, \quad B_3 = \sum_{i=1}^{5}\sum_{m=1}^{2} d_i b_m J_{n_i+l_m}, \quad B_5 = -\hat{J}_2,$$

$$B_4 = \sum_{k=1}^{5}\sum_{m=1}^{5} d_k(a_m J_{n_k+l_m} + c_m \bar{L}_{n_k+l_m}) + \sum_{i=1}^{5}\sum_{j=1}^{5*}\sum_{k=1}^{5} d_i d_j d_k I_{n_i+n_j+1} J_{n_i+n_j+n_k+1},$$

$$J_q = \int_{\xi}^{1} \sum_{i=1}^{5} d_i \rho^{n_i}\rho^q d\rho = \sum_{i=1}^{5} d_i H_{n_i+q}, \quad \text{with} \quad H_m = \int_{\xi}^{1} \rho^m \,\mathrm{d}\rho,$$

$$H_m = (1-\xi^{m+1})/(m+1) \quad \text{for } m \neq -1 \quad \text{and} \quad H_{-1} = -\ln\xi,$$

$$\hat{J}_q = (J_q - \xi^q J_0)/q, \quad \bar{L}_q = \int_{\xi}^{1} \sum_{i=1}^{5} d_i \rho^{n_i}\rho^q \ln\rho\,\mathrm{d}\rho = \sum_{i=1}^{5} d_i L_{n_i+q}, \quad L_m = \int_{\xi}^{1} \rho^m \ln\rho\,\mathrm{d}\rho,$$

$$L_m = -[1 + \{(m+1)\ln\xi - 1\}\xi^{m+1}]/(\mathrm{m}+1)^2 \quad \text{for } m \neq -1 \text{ and } L_{-1} = -(\ln\xi)^2/2. \tag{15}$$

The deflection at the inner edge is given by $w(\xi,\tau) = \eta_1(\tau) = B_6\eta(\tau)$ with $B_6 = \sum_{i=1}^{5} w_i \xi^{m_i} + w_6 \ln\xi$. The governing equation for deflection at the inner edge is obtained from equation (14) as

$$A_1\ddot{\eta}_1 + (A_2 + A_3 N)\eta_1 + A_4\eta_1^3 = A_5 Q, \tag{16}$$

where $A_1 = B_1/B_6$, $A_2 = B_2/B_6$, $A_3 = B_3/B_6$, $A_4 = B_4/(B_6)^3$, $A_5 = B_5$.

The buckling load N_{cr} and the postbuckling response is obtained by setting $\ddot{\eta}_1 = 0$, $Q = 0$ in equation (16):

$$n = N/N_{cr} = 1 + \varepsilon_1\eta_1^2, \qquad N_{cr} = -A_2/A_3, \quad \varepsilon_1 = A_4/A_2, \tag{17}$$

with ε_1 being the postbuckling parameter. The static response under uniformly distributed load Q is given by

$$Q = \alpha_1(1-n)\eta_1 + \alpha_2\eta_1^3 = \alpha_1\eta_1[1 - n + \varepsilon_1\eta_1^2], \qquad \alpha_1 = A_2/A_5, \qquad \alpha_2 = A_4/A_5. \tag{18}$$

The maximum deflection $\eta_{1\,max}$ under step load Q_0 is obtained by integrating equation (16):

$$Q_0 = \alpha_1(1-n)\eta_{1_{max}}/2 + \alpha_2\eta_{1_{max}}^3/4 = \alpha_1[1 - n + \varepsilon_1\eta_{1_{max}}^2/2]\eta_{1_{max}}/2. \tag{19}$$

The free non-linear vibrations are governed by

$$\ddot{\eta}_1 + \omega_0^2(\eta_1 + \varepsilon\eta_1^3) = 0, \qquad \omega_0 = \omega_0^*[\gamma a^4/E_0 h^2]^{1/2} = [(1-n)A_2/A_1]^{1/2}, \quad \varepsilon = \varepsilon_1/(1-n), \tag{20}$$

where ω_0 and ω_0^* are the dimensionless and dimensional fundamental linear frequencies. The ratio T_C of the non-linear period T for amplitude C to the linear period T_0 can be expressed in terms of the complete elliptic integral K of the first kind:

$$T_C = T/T_0 = 2K(e)/\pi(1+\varepsilon C^2)^{1/2}, \quad e = [\varepsilon C^2/2(1+\varepsilon C^2)]^{1/2}. \tag{21}$$

TABLE 1

Comparison of postbuckling response ($b/a = 0{\cdot}4$)

		$-N$											
		CM						SM					
		Present			Collocation			Present			Collocation		
β	s	$\eta_1 = 0$	$\eta_1 = 1$	$\eta_1 = 2$	$\eta_1 = 0$	$\eta_1 = 1$	$\eta_1 = 2$	$\eta_1 = 0$	$\eta_1 = 1$	$\eta_1 = 2$	$\eta_1 = 0$	$\eta_1 = 1$	$\eta_1 = 2$
1	100	1·803	2·020	2·672	1·690	1·941	2·653	0·250	0·356	0·674	0·248	0·353	0·649
	10	1·773	1·980	2·599	1·632	1·879	2·572	0·249	0·355	0·671	0·246	0·351	0·644
	6	1·706	1·896	2·464	1·536	1·776	2·436	0·246	0·351	0·666	0·243	0·347	0·635
3	100	2·961	3·538	5·268	2·822	3·485	5·267	0·741	1·007	1·807	0·741	1·001	1·729
	10	2·852	3·393	5·015	2·675	3·318	5·003	0·728	0·993	1·788	0·728	0·986	1·696
	6	2·643	3·130	4·593	2·444	3·052	4·575	0·705	0·968	1·758	0·705	0·959	1·640
10	100	7·413	8·980	13·679	7·315	8·957	13·559	2·386	3·096	5·223	2·348	3·027	5·070
	10	6·675	8·105	12·392	6·498	8·026	12·104	2·271	2·979	5·103	2·227	2·891	4·816
	6	5·549	6·803	10·564	5·375	6·716	9·902	2·089	2·795	4·915	2·038	2·675	4·402
1/3	100	4·388	4·607	5·285	4·120	4·375	5·125	0·230	0·337	0·660	0·230	0·337	0·652
	10	4·198	4·391	4·970	3·774	4·022	4·741	0·229	0·336	0·658	0·228	0·336	0·647
	6	3·737	3·898	4·380	3·272	3·508	4·176	0·226	0·333	0·653	0·226	0·333	0·638
		$k_b = 1$, $k_i = 0$						$k_b = k_i = 1$					
1/3	100	2·053	2·193	2·627	2·012	2·169	2·624	4·328	5·052	7·225	4·242	4·980	7·191
	10	1·966	2·103	2·515	1·918	2·072	2·515	4·134	4·865	7·026	4·044	4·783	6·994
	6	1·821	1·950	2·335	1·769	1·918	2·338	3·839	4·557	6·710	3·729	4·470	6·686

4. RESULTS AND CONCLUSIONS

The results are non-dimensionalized with E_0 taken to be equal to the smaller in-plane Young's modulus. Plates are analyzed with $\beta = E_\theta/E_r = 1, 3, 10, 1/3$, having the greater in-plane Poisson ratio of 0·25 and $G_{rz}/E_0 = 0{\cdot}4$. The buckling and postbuckling loads of thick polar orthotropic annular plate with $b/a = 0{\cdot}4$ are compared in Table 1 with those obtained using orthogonal point collocation method of reference [6]. All the present results for the isotropic case are obtained using $\beta = 1{\cdot}01$. In most cases, the error is less than 5% for orthotropic plates and it generally increases with s and decreases with β. The buckling loads of CM plates are less accurate, but the postbuckling loads for $\eta_1 = 2$ are quite accurate. The non-linear vibration response of reference [1] using FEM, for isotropic thick circular plate, presented in Table 2, is in excellent agreement with the present Galerkin solution for a slightly orthotropic plate with a small hole with $\beta = 1{\cdot}01$, $b/a = 0{\cdot}01$. The maximum deflection of plates with $b/a = 0{\cdot}5$ under step load $Q_0 = 15$ is compared in Table 3 with the collocation solution of reference [5]. There is very good agreement among these solutions with the maximum difference being 5%. The corresponding static response, not reported here for brevity, is also in good agreement.

The response parameters of clamped and simply supported thick plates with $b/a = 0{\cdot}3$ are given in Table 4. The critical load N_{cr} is for the movable inplane condition. The non-linear parameter for the immovable and movable inplane conditions is listed as $\varepsilon_1(I)$ and $\varepsilon_1(M)$ respectively. It is observed that the non-linearity parameter ε_1 is much more for the simply supported plates than the clamped ones. It is also much more for the immovable in-plane condition that the movable one. The influence of the thickness parameter s on ε_1 increases with the modular ratio β. The effect of s on α_1 N_{cr}, ω_0 is much more for the clamped plates than the simply supported ones and this effect increases with β.

The dependence of the non-linear response parameter ε_1, for plate with $b/a = 0{\cdot}3$, $\beta = 3$ on the in-plane support stiffness k_i, for the clamped ($k_b = \infty$) and the simply supported ($k_b = 0$) cases, is shown in Figure 1(a). The dependence of ε_1 for these plates on the rotational support stiffness k_b, for the immovable ($k_i = \infty$) inplane condition, is also shown in Figure 1(a). The rate of increase of ε_1 with k_i is predominant in the range of $k_i = 0{\cdot}1$–10. The rate of decrease of ε_1 with k_b is predominant in the range of $k_b = 0{\cdot}2$–2. The dependence of ε_1 for plates with $b/a = 0{\cdot}3$, $\beta = 3$ on the rotational support stiffness k_b, for the movable ($k_i = 0$) in-plane condition and for the case of $k_i = k_b$, are presented in Figure 1(b). The effect of the rotational stiffness k_b of movable support on N_{cr}, α_1, ω_0 for these plates is shown in Figure 2. The rate of change of these response parameters is significant in the intermediate range of k_b.

TABLE 2

Comparison of non-linear vibration response ($v_\theta = 0{\cdot}3$)

	SI				CI			
	Present		Reference [1] (FEM)		Present		Reference [1] (FEM)	
s	ω_0	T_1/T_0	ω_0	T_1/T_0	ω_0	T_1/T_0	ω_0	T_1/T_0
1000	1·490	0·6517	1·494	0·6682	3·119	0·8612	3·091	0·8607
10	1·482	0·6494	1·481	0·6670	3·050	0·8598	3·011	0·8591
5	1·457	0·6424	1·446	0·6634	2·863	0·8532	2·806	0·8533

TABLE 3

Maximum deflection at the hole under step load $Q_0 = 15$ ($b/a = 0{\cdot}5$)

	w_{max}													
	CI								CM		SM		$k_b = k_i = 1$	
	$\beta = 1$		$\beta = 3$		$\beta = 10$		$\beta = 1/3$		$\beta = 10$		$\beta = 10$		$\beta = 1/3$	
s	Gal.	Coll.	Gal.	Coll.	Gal.	Coll.	Gal.	Coll.	Gal.	Coll.	Gal.	Coll.	Gal.	Coll.
100	1·429	1·423	1·115	1·119	0·714	0·729	0·625	0·626	0·740	0·755	1·569	1·560	1·537	1·544
10	1·482	1·496	1·164	1·153	0·777	0·774	0·710	0·730	0·815	0·816	1·596	1·594	1·589	1·570
6	1·567	1·530	1·242	1·171	0·874	0·850	0·855	0·855	0·941	0·899	1·640	1·620	1·673	1·684

TABLE 4

Response parameters of annular plates ($b/a = 0{\cdot}3$)

		Clamped					Simply supported				
β	s	α_1	ω_0	$-N_{cr}(M)$	$\varepsilon_1(I)$	$\varepsilon_1(M)$	α_1	ω_0	$-N_{cr}(M)$	$\varepsilon_1(I)$	$\varepsilon_1(M)$
1	100	6·947	3·478	1·480	0·3898	0·1506	1·184	1·398	0·279	1·938	0·3788
	10	6·707	3·413	1·459	0·3904	0·1462	1·177	1·393	0·277	1·956	0·3803
	6	6·316	3·302	1·412	0·3962	0·1403	1·165	1·384	0·274	1·987	0·3829
3	100	11·003	4·251	2·824	0·4131	0·1969	3·242	2·234	0·792	1·414	0·3207
	10	10·414	4·127	2·713	0·4226	0·1939	3·188	2·213	0·776	1·447	0·3261
	6	9·499	3·925	2·509	0·4472	0·1919	3·097	2·178	0·750	1·508	0·3359
10	100	22·222	5·743	7·509	0·3310	0·2006	9·552	3·656	2·441	0·968	0·2827
	10	19·941	5·433	6·746	0·3554	0·2057	9·104	3·568	2·324	1·018	0·2965
	6	16·837	4·977	5·609	0·4101	0·2195	8·398	3·425	2·139	1·107	0·3211
1/3	100	16·586	5·398	3·199	0·2004	0·0727	1·122	1·370	0·252	2·349	0·4462
	10	15·281	5·165	3·107	0·2004	0·0675	1·115	1·365	0·250	2·368	0·4475
	6	13·390	4·795	2·862	0·2121	0·0626	1·104	1·357	0·247	2·403	0·4498

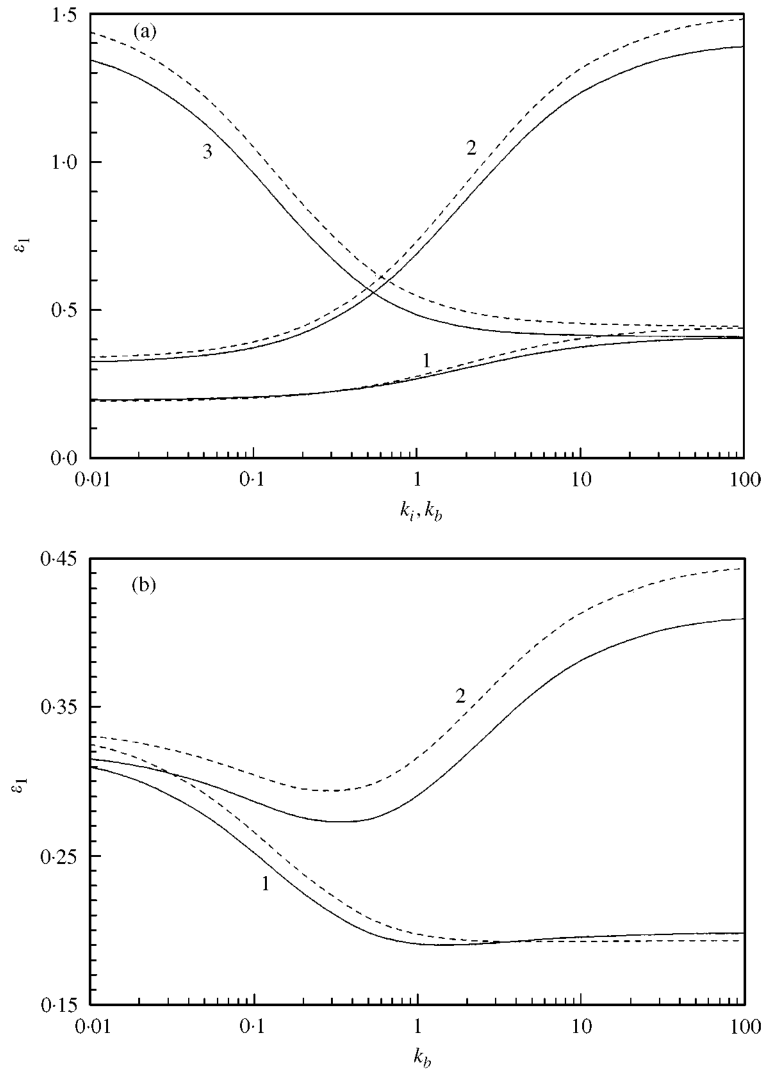

Figure 1. Variation of non-linearity parameter ε_1 for plates with $b/a = 0{\cdot}3$, $\beta = 3$, (a) with k_i, k_b: (1) $k_b = \infty$, (2) $k_b = 0$, (3) $k_i = \infty$ and (b) with k_b: (1) $k_i = 0$, (2) $k_i = k_b$; (——), $S = 100$; (– – –), $S = 6$.

It is concluded that the present unified simple approximate analytical solution for non-linear static, vibratory and transient response of thick orthotropic annular plate under axisymmetric transverse and in-plane loads yields accurate results.

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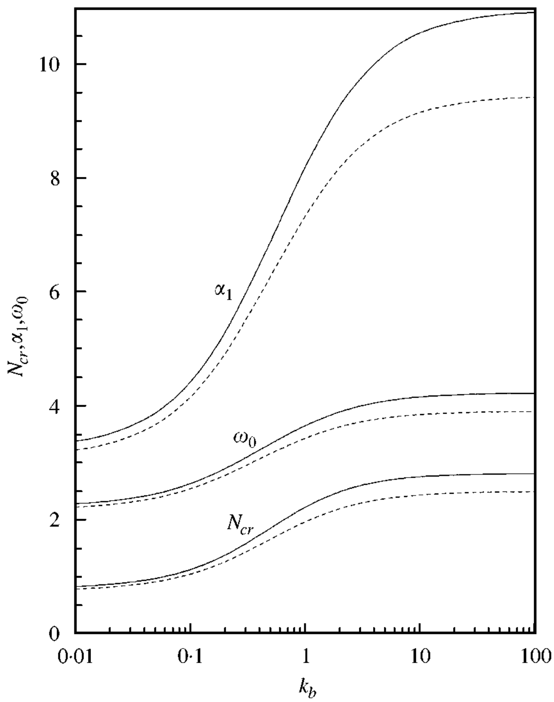

Figure 2. The effect of rotational stiffness k_b of a movable support on N_{cr}, α_1, ω_0. (——), $S = 100$; (- - -), $S = 6$.

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